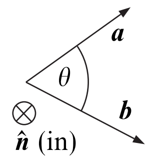
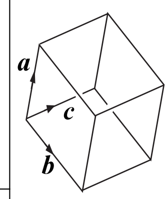
2.2 Vectors and matrices

Vector algebra

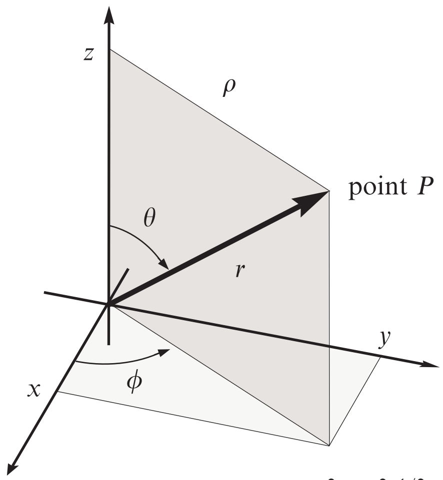
Scalar product[a]	$\boldsymbol{a}\cdot\boldsymbol{b}=\lvert\boldsymbol{a}\rvert\lvert\boldsymbol{b}\rvert\cos\theta$	(2.1)
Vector product[b]	$\boldsymbol{a}\times\boldsymbol{b}=\lvert\boldsymbol{a}\rvert\lvert\boldsymbol{b}\rvert\sin\theta\,\hat{\boldsymbol{n}}=\begin{vmatrix}\hat{\boldsymbol{x}} & \hat{\boldsymbol{y}} & \hat{\boldsymbol{z}}\\ a_x & a_y & a_z\\ b_x & b_y & b_z\end{vmatrix}$	(2.2)
Product rules	$\boldsymbol{a}\cdot\boldsymbol{b}=\boldsymbol{b}\cdot\boldsymbol{a}$	(2.3)
	$\boldsymbol{a}\times\boldsymbol{b}=-\boldsymbol{b}\times\boldsymbol{a}$	(2.4)
	$\boldsymbol{a}\cdot(\boldsymbol{b}+\boldsymbol{c})=(\boldsymbol{a}\cdot\boldsymbol{b})+(\boldsymbol{a}\cdot\boldsymbol{c})$	(2.5)
	$\boldsymbol{a}\times(\boldsymbol{b}+\boldsymbol{c})=(\boldsymbol{a}\times\boldsymbol{b})+(\boldsymbol{a}\times\boldsymbol{c})$	(2.6)
Lagrange's identity	$(\boldsymbol{a}\times\boldsymbol{b})\cdot(\boldsymbol{c}\times\boldsymbol{d})=(\boldsymbol{a}\cdot\boldsymbol{c})(\boldsymbol{b}\cdot\boldsymbol{d})-(\boldsymbol{a}\cdot\boldsymbol{d})(\boldsymbol{b}\cdot\boldsymbol{c})$	(2.7)
Scalar triple product	$(\boldsymbol{a}\times\boldsymbol{b})\cdot\boldsymbol{c}=\begin{vmatrix}a_x & a_y & a_z\\ b_x & b_y & b_z\\ c_x & c_y & c_z\end{vmatrix}$	(2.8)
	$=(\boldsymbol{b}\times\boldsymbol{c})\cdot\boldsymbol{a}=(\boldsymbol{c}\times\boldsymbol{a})\cdot\boldsymbol{b}$	(2.9)
	$=$ volume of parallelepiped	(2.10)
Vector triple product	$(\boldsymbol{a}\times\boldsymbol{b})\times\boldsymbol{c}=(\boldsymbol{a}\cdot\boldsymbol{c})\boldsymbol{b}-(\boldsymbol{b}\cdot\boldsymbol{c})\boldsymbol{a}$	(2.11)
	$\boldsymbol{a}\times(\boldsymbol{b}\times\boldsymbol{c})=(\boldsymbol{a}\cdot\boldsymbol{c})\boldsymbol{b}-(\boldsymbol{a}\cdot\boldsymbol{b})\boldsymbol{c}$	(2.12)
Reciprocal vectors	$\boldsymbol{a}'=(\boldsymbol{b}\times\boldsymbol{c})/[(\boldsymbol{a}\times\boldsymbol{b})\cdot\boldsymbol{c}]$	(2.13)
	$\boldsymbol{b}'=(\boldsymbol{c}\times\boldsymbol{a})/[(\boldsymbol{a}\times\boldsymbol{b})\cdot\boldsymbol{c}]$	(2.14)
	$\boldsymbol{c}'=(\boldsymbol{a}\times\boldsymbol{b})/[(\boldsymbol{a}\times\boldsymbol{b})\cdot\boldsymbol{c}]$	(2.15)
	$(\boldsymbol{a}'\cdot\boldsymbol{a})=(\boldsymbol{b}'\cdot\boldsymbol{b})=(\boldsymbol{c}'\cdot\boldsymbol{c})=1$	(2.16)
Vector $\boldsymbol{a}$ with respect to a nonorthogonal basis $\{\boldsymbol{e}_1,\boldsymbol{e}_2,\boldsymbol{e}_3\}$[c]	$\boldsymbol{a}=(\boldsymbol{e}_1'\cdot\boldsymbol{a})\boldsymbol{e}_1+(\boldsymbol{e}_2'\cdot\boldsymbol{a})\boldsymbol{e}_2+(\boldsymbol{e}_3'\cdot\boldsymbol{a})\boldsymbol{e}_3$	(2.17)





[a] Also known as the "dot product" or the "inner product."
[b] Also known as the "cross-product." $\hat{\boldsymbol{n}}$ is a unit vector making a right-handed set with $\boldsymbol{a}$ and $\boldsymbol{b}$.
[c] The prime (′) denotes a reciprocal vector.

Common three-dimensional coordinate systems



$$x = \rho\cos\phi = r\sin\theta\cos\phi \quad (2.18)$$

$$y = \rho\sin\phi = r\sin\theta\sin\phi \quad (2.19)$$

$$z = r\cos\theta \quad (2.20)$$

$$\rho = (x^2+y^2)^{1/2} \quad (2.21)$$

$$r = (x^2+y^2+z^2)^{1/2} \quad (2.22)$$

$$\theta = \arccos(z/r) \quad (2.23)$$

$$\phi = \arctan(y/x) \quad (2.24)$$

coordinate system:	rectangular	spherical polar	cylindrical polar
coordinates of P:	(x,y,z)	(r,θ,ϕ)	(ρ,ϕ,z)
volume element:	$\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$	$r^2\sin\theta\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}\phi$	$\rho\,\mathrm{d}\rho\,\mathrm{d}z\,\mathrm{d}\phi$
metric elements[a] (h_1,h_2,h_3):	$(1,1,1)$	$(1,r,r\sin\theta)$	$(1,\rho,1)$

[a] In an orthogonal coordinate system (parameterised by coordinates q_1,q_2,q_3), the differential line element $\mathrm{d}l$ is obtained from $(\mathrm{d}l)^2 = (h_1\,\mathrm{d}q_1)^2+(h_2\,\mathrm{d}q_2)^2+(h_3\,\mathrm{d}q_3)^2$.

Gradient

Rectangular coordinates	$\nabla f = \frac{\partial f}{\partial x}\hat{\boldsymbol{x}} + \frac{\partial f}{\partial y}\hat{\boldsymbol{y}} + \frac{\partial f}{\partial z}\hat{\boldsymbol{z}}$	(2.25)	f scalar field $\hat{}$ unit vector
Cylindrical coordinates	$\nabla f = \frac{\partial f}{\partial \rho}\hat{\boldsymbol{\rho}} + \frac{1}{r}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\phi}} + \frac{\partial f}{\partial z}\hat{\boldsymbol{z}}$	(2.26)	ρ distance from the z axis
Spherical polar coordinates	$\nabla f = \frac{\partial f}{\partial r}\hat{\boldsymbol{r}} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\phi}}$	(2.27)	
General orthogonal coordinates	$\nabla f = \frac{\hat{\boldsymbol{q}}_1}{h_1}\frac{\partial f}{\partial q_1} + \frac{\hat{\boldsymbol{q}}_2}{h_2}\frac{\partial f}{\partial q_2} + \frac{\hat{\boldsymbol{q}}_3}{h_3}\frac{\partial f}{\partial q_3}$	(2.28)	q_i basis h_i metric elements

Divergence

Rectangular coordinates	$\nabla \cdot \boldsymbol{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$	(2.29)	$\boldsymbol{A}$ A_i	vector field ith component of $\boldsymbol{A}$
Cylindrical coordinates	$\nabla \cdot \boldsymbol{A} = \frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial \rho} + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$	(2.30)	ρ	distance from the z axis
Spherical polar coordinates	$\nabla \cdot \boldsymbol{A} = \frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(A_\theta \sin\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$	(2.31)		
General orthogonal coordinates	$\nabla \cdot \boldsymbol{A} = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial q_1}(A_1 h_2 h_3) + \frac{\partial}{\partial q_2}(A_2 h_3 h_1) + \frac{\partial}{\partial q_3}(A_3 h_1 h_2)\right]$	(2.32)	q_i h_i	basis metric elements

Curl

Rectangular coordinates	$\nabla \times \boldsymbol{A} = \begin{vmatrix} \hat{\boldsymbol{x}} & \hat{\boldsymbol{y}} & \hat{\boldsymbol{z}} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{vmatrix}$	(2.33)	$\hat{}$ $\boldsymbol{A}$ A_i	unit vector vector field ith component of $\boldsymbol{A}$
Cylindrical coordinates	$\nabla \times \boldsymbol{A} = \begin{vmatrix} \hat{\boldsymbol{\rho}}/\rho & \hat{\boldsymbol{\phi}} & \hat{\boldsymbol{z}}/\rho \\ \partial/\partial \rho & \partial/\partial \phi & \partial/\partial z \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$	(2.34)	ρ	distance from the z axis
Spherical polar coordinates	$\nabla \times \boldsymbol{A} = \begin{vmatrix} \hat{\boldsymbol{r}}/(r^2\sin\theta) & \hat{\boldsymbol{\theta}}/(r\sin\theta) & \hat{\boldsymbol{\phi}}/r \\ \partial/\partial r & \partial/\partial \theta & \partial/\partial \phi \\ A_r & rA_\theta & rA_\phi \sin\theta \end{vmatrix}$	(2.35)		
General orthogonal coordinates	$\nabla \times \boldsymbol{A} = \frac{1}{h_1 h_2 h_3}\begin{vmatrix} \hat{\boldsymbol{q}}_1 h_1 & \hat{\boldsymbol{q}}_2 h_2 & \hat{\boldsymbol{q}}_3 h_3 \\ \partial/\partial q_1 & \partial/\partial q_2 & \partial/\partial q_3 \\ h_1 A_1 & h_2 A_2 & h_3 A_3 \end{vmatrix}$	(2.36)	q_i h_i	basis metric elements

Radial forms[a]

$\nabla r = \frac{\boldsymbol{r}}{r}$	(2.37)	$\nabla(1/r) = \frac{-\boldsymbol{r}}{r^3}$	(2.41)
$\nabla \cdot \boldsymbol{r} = 3$	(2.38)	$\nabla \cdot (\boldsymbol{r}/r^2) = \frac{1}{r^2}$	(2.42)
$\nabla r^2 = 2\boldsymbol{r}$	(2.39)	$\nabla(1/r^2) = \frac{-2\boldsymbol{r}}{r^4}$	(2.43)
$\nabla \cdot (r\boldsymbol{r}) = 4r$	(2.40)	$\nabla \cdot (\boldsymbol{r}/r^3) = 4\pi\delta(\boldsymbol{r})$	(2.44)

[a]Note that the curl of any purely radial function is zero. $\delta(\boldsymbol{r})$ is the Dirac delta function.

Laplacian (scalar)

Rectangular coordinates	$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} \quad (2.45)$$	f scalar field
Cylindrical coordinates	$$\nabla^2 f = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial f}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2} \quad (2.46)$$	ρ distance from the z axis
Spherical polar coordinates	$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial f}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 f}{\partial \phi^2} \quad (2.47)$$	
General orthogonal coordinates	$$\nabla^2 f = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial q_1}\left(\frac{h_2 h_3}{h_1}\frac{\partial f}{\partial q_1}\right) + \frac{\partial}{\partial q_2}\left(\frac{h_3 h_1}{h_2}\frac{\partial f}{\partial q_2}\right) + \frac{\partial}{\partial q_3}\left(\frac{h_1 h_2}{h_3}\frac{\partial f}{\partial q_3}\right)\right] \quad (2.48)$$	q_i basis h_i metric elements

Differential operator identities

$$\nabla(fg) \equiv f\nabla g + g\nabla f \quad (2.49)$$	
$$\nabla \cdot (f\boldsymbol{A}) \equiv f\nabla \cdot \boldsymbol{A} + \boldsymbol{A} \cdot \nabla f \quad (2.50)$$	
$$\nabla \times (f\boldsymbol{A}) \equiv f\nabla \times \boldsymbol{A} + (\nabla f) \times \boldsymbol{A} \quad (2.51)$$	
$$\nabla(\boldsymbol{A} \cdot \boldsymbol{B}) \equiv \boldsymbol{A} \times (\nabla \times \boldsymbol{B}) + (\boldsymbol{A} \cdot \nabla)\boldsymbol{B} + \boldsymbol{B} \times (\nabla \times \boldsymbol{A}) + (\boldsymbol{B} \cdot \nabla)\boldsymbol{A} \quad (2.52)$$	
$$\nabla \cdot (\boldsymbol{A} \times \boldsymbol{B}) \equiv \boldsymbol{B} \cdot (\nabla \times \boldsymbol{A}) - \boldsymbol{A} \cdot (\nabla \times \boldsymbol{B}) \quad (2.53)$$	f,g scalar fields
$$\nabla \times (\boldsymbol{A} \times \boldsymbol{B}) \equiv \boldsymbol{A}(\nabla \cdot \boldsymbol{B}) - \boldsymbol{B}(\nabla \cdot \boldsymbol{A}) + (\boldsymbol{B} \cdot \nabla)\boldsymbol{A} - (\boldsymbol{A} \cdot \nabla)\boldsymbol{B} \quad (2.54)$$	$\boldsymbol{A},\boldsymbol{B}$ vector fields
$$\nabla \cdot (\nabla f) \equiv \nabla^2 f \equiv \triangle f \quad (2.55)$$	
$$\nabla \times (\nabla f) \equiv \boldsymbol{0} \quad (2.56)$$	
$$\nabla \cdot (\nabla \times \boldsymbol{A}) \equiv 0 \quad (2.57)$$	
$$\nabla \times (\nabla \times \boldsymbol{A}) \equiv \nabla(\nabla \cdot \boldsymbol{A}) - \nabla^2 \boldsymbol{A} \quad (2.58)$$	

Vector integral transformations

Gauss's (Divergence) theorem	$$\int_V (\nabla \cdot \boldsymbol{A})\,\mathrm{d}V = \oint_{S_c} \boldsymbol{A} \cdot \mathrm{d}\boldsymbol{s} \quad (2.59)$$	$\boldsymbol{A}$ vector field $\mathrm{d}V$ volume element S_c closed surface V volume enclosed
Stokes's theorem	$$\int_S (\nabla \times \boldsymbol{A}) \cdot \mathrm{d}\boldsymbol{s} = \oint_L \boldsymbol{A} \cdot \mathrm{d}\boldsymbol{l} \quad (2.60)$$	S surface $\mathrm{d}\boldsymbol{s}$ surface element L loop bounding S $\mathrm{d}\boldsymbol{l}$ line element
Green's first theorem	$$\oint_S (f\nabla g) \cdot \mathrm{d}\boldsymbol{s} = \int_V \nabla \cdot (f\nabla g)\,\mathrm{d}V \quad (2.61)$$ $$= \int_V [f\nabla^2 g + (\nabla f) \cdot (\nabla g)]\,\mathrm{d}V \quad (2.62)$$	f,g scalar fields
Green's second theorem	$$\oint_S [f(\nabla g) - g(\nabla f)] \cdot \mathrm{d}\boldsymbol{s} = \int_V (f\nabla^2 g - g\nabla^2 f)\,\mathrm{d}V \quad (2.63)$$	

Matrix algebra[a]

Matrix definition	$\mathbf{A}=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$	(2.64)	$\mathbf{A}$ m by n matrix a_{ij} matrix elements
Matrix addition	$\mathbf{C}=\mathbf{A}+\mathbf{B}$ if $c_{ij}=a_{ij}+b_{ij}$	(2.65)	
Matrix multiplication	$\mathbf{C}=\mathbf{AB}$ if $c_{ij}=a_{ik}b_{kj}$ $(\mathbf{AB})\mathbf{C}=\mathbf{A}(\mathbf{BC})$ $\mathbf{A}(\mathbf{B}+\mathbf{C})=\mathbf{AB}+\mathbf{AC}$	(2.66) (2.67) (2.68)	
Transpose matrix[b]	$\tilde{a}_{ij}=a_{ji}$ $(\widetilde{\mathbf{AB}\ldots\mathbf{N}})=\tilde{\mathbf{N}}\ldots\tilde{\mathbf{B}}\tilde{\mathbf{A}}$	(2.69) (2.70)	$\tilde{a}_{ij}$ transpose matrix (sometimes a^{T}_{ij}, or a'_{ij})
Adjoint matrix (definition 1)[c]	$\mathbf{A}^{\dagger}=\tilde{\mathbf{A}}^{*}$ $(\mathbf{AB}\ldots\mathbf{N})^{\dagger}=\mathbf{N}^{\dagger}\ldots\mathbf{B}^{\dagger}\mathbf{A}^{\dagger}$	(2.71) (2.72)	* complex conjugate (of each component) † adjoint (or Hermitian conjugate)
Hermitian matrix[d]	$\mathbf{H}^{\dagger}=\mathbf{H}$	(2.73)	$\mathbf{H}$ Hermitian (or self-adjoint) matrix

examples:

$$\mathbf{A}=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \qquad \mathbf{B}=\begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}$$

$$\tilde{\mathbf{A}}=\begin{pmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{pmatrix} \qquad \mathbf{A}+\mathbf{B}=\begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} & a_{13}+b_{13} \\ a_{21}+b_{21} & a_{22}+b_{22} & a_{23}+b_{23} \\ a_{31}+b_{31} & a_{32}+b_{32} & a_{33}+b_{33} \end{pmatrix}$$

$$\mathbf{AB}=\begin{pmatrix} a_{11}b_{11}+a_{12}b_{21}+a_{13}b_{31} & a_{11}b_{12}+a_{12}b_{22}+a_{13}b_{32} & a_{11}b_{13}+a_{12}b_{23}+a_{13}b_{33} \\ a_{21}b_{11}+a_{22}b_{21}+a_{23}b_{31} & a_{21}b_{12}+a_{22}b_{22}+a_{23}b_{32} & a_{21}b_{13}+a_{22}b_{23}+a_{23}b_{33} \\ a_{31}b_{11}+a_{32}b_{21}+a_{33}b_{31} & a_{31}b_{12}+a_{32}b_{22}+a_{33}b_{32} & a_{31}b_{13}+a_{32}b_{23}+a_{33}b_{33} \end{pmatrix}$$

[a]Terms are implicitly summed over repeated suffices; hence $a_{ik}b_{kj}$ equals $\sum_k a_{ik}b_{kj}$.
[b]See also Equation (2.85).
[c]Or "Hermitian conjugate matrix." The term "adjoint" is used in quantum physics for the transpose conjugate of a matrix and in linear algebra for the transpose matrix of its cofactors. These definitions are not compatible, but both are widely used [cf. Equation (2.80)].
[d]Hermitian matrices must also be square (see next table).

Square matrices[a]

Trace	$\mathrm{tr}\,\mathbf{A}=a_{ii}$ (2.74) $\mathrm{tr}(\mathbf{AB})=\mathrm{tr}(\mathbf{BA})$ (2.75)	$\mathbf{A}$ a_{ij} a_{ii}	square matrix matrix elements implicitly $=\sum_i a_{ii}$
Determinant[b]	$\det\mathbf{A}=\epsilon_{ijk\ldots}a_{1i}a_{2j}a_{3k}\ldots$ (2.76) $=(-1)^{i+1}a_{i1}M_{i1}$ (2.77) $=a_{i1}C_{i1}$ (2.78) $\det(\mathbf{AB}\ldots\mathbf{N})=\det\mathbf{A}\det\mathbf{B}\ldots\det\mathbf{N}$ (2.79)	tr det M_{ij} C_{ij}	trace determinant (or $\lvert\mathbf{A}\rvert$) minor of element a_{ij} cofactor of the element a_{ij}
Adjoint matrix (definition 2)[c]	$\mathrm{adj}\,\mathbf{A}=\tilde{C}_{ij}=C_{ji}$ (2.80)	adj $\sim$	adjoint (sometimes written $\hat{\mathbf{A}}$) transpose
Inverse matrix ($\det\mathbf{A}\neq 0$)	$a_{ij}^{-1}=\dfrac{C_{ji}}{\det\mathbf{A}}=\dfrac{\mathrm{adj}\,\mathbf{A}}{\det\mathbf{A}}$ (2.81) $\mathbf{A}\mathbf{A}^{-1}=\mathbf{1}$ (2.82) $(\mathbf{AB}\ldots\mathbf{N})^{-1}=\mathbf{N}^{-1}\ldots\mathbf{B}^{-1}\mathbf{A}^{-1}$ (2.83)	$\mathbf{1}$	unit matrix
Orthogonality condition	$a_{ij}a_{ik}=\delta_{jk}$ (2.84) i.e., $\tilde{\mathbf{A}}=\mathbf{A}^{-1}$ (2.85)	δ_{jk}	Kronecker delta ($=1$ if $i=j$, $=0$ otherwise)
Symmetry	If $\mathbf{A}=\tilde{\mathbf{A}}$, $\mathbf{A}$ is symmetric (2.86) If $\mathbf{A}=-\tilde{\mathbf{A}}$, $\mathbf{A}$ is antisymmetric (2.87)		
Unitary matrix	$\mathbf{U}^{\dagger}=\mathbf{U}^{-1}$ (2.88)	$\mathbf{U}$ $\dagger$	unitary matrix Hermitian conjugate

examples:

$$\mathbf{A}=\begin{pmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{pmatrix}\qquad \mathbf{B}=\begin{pmatrix}b_{11} & b_{12}\\ b_{21} & b_{22}\end{pmatrix}$$

$$\mathrm{tr}\,\mathbf{A}=a_{11}+a_{22}+a_{33}\qquad \mathrm{tr}\,\mathbf{B}=b_{11}+b_{22}$$

$$\det\mathbf{A}=a_{11}a_{22}a_{33}-a_{11}a_{23}a_{32}-a_{21}a_{12}a_{33}+a_{21}a_{13}a_{32}+a_{31}a_{12}a_{23}-a_{31}a_{13}a_{22}$$

$$\det\mathbf{B}=b_{11}b_{22}-b_{12}b_{21}$$

$$\mathbf{A}^{-1}=\frac{1}{\det\mathbf{A}}\begin{pmatrix}a_{22}a_{33}-a_{23}a_{32} & -a_{12}a_{33}+a_{13}a_{32} & a_{12}a_{23}-a_{13}a_{22}\\ -a_{21}a_{33}+a_{23}a_{31} & a_{11}a_{33}-a_{13}a_{31} & -a_{11}a_{23}+a_{13}a_{21}\\ a_{21}a_{32}-a_{22}a_{31} & -a_{11}a_{32}+a_{12}a_{31} & a_{11}a_{22}-a_{12}a_{21}\end{pmatrix}$$

$$\mathbf{B}^{-1}=\frac{1}{\det\mathbf{B}}\begin{pmatrix}b_{22} & -b_{12}\\ -b_{21} & b_{11}\end{pmatrix}$$

[a] Terms are implicitly summed over repeated suffices; hence $a_{ik}b_{kj}$ equals $\sum_k a_{ik}b_{kj}$.
[b] $\epsilon_{ijk\ldots}$ is defined as the natural extension of Equation (2.443) to n-dimensions (see page 50). M_{ij} is the determinant of the matrix $\mathbf{A}$ with the ith row and the jth column deleted. The cofactor $C_{ij}=(-1)^{i+j}M_{ij}$.
[c] Or "adjugate matrix." See the footnote to Equation (2.71) for a discussion of the term "adjoint."

Commutators

Commutator definition	$[\mathbf{A},\mathbf{B}]=\mathbf{AB}-\mathbf{BA}=-[\mathbf{B},\mathbf{A}]$	(2.89)	$[\cdot,\cdot]$	commutator
Adjoint	$[\mathbf{A},\mathbf{B}]^\dagger=[\mathbf{B}^\dagger,\mathbf{A}^\dagger]$	(2.90)	$\dagger$	adjoint
Distribution	$[\mathbf{A}+\mathbf{B},\mathbf{C}]=[\mathbf{A},\mathbf{C}]+[\mathbf{B},\mathbf{C}]$	(2.91)		
Association	$[\mathbf{AB},\mathbf{C}]=\mathbf{A}[\mathbf{B},\mathbf{C}]+[\mathbf{A},\mathbf{C}]\mathbf{B}$	(2.92)		
Jacobi identity	$[\mathbf{A},[\mathbf{B},\mathbf{C}]]=[\mathbf{B},[\mathbf{A},\mathbf{C}]]-[\mathbf{C},[\mathbf{A},\mathbf{B}]]$	(2.93)		

Pauli matrices

Pauli matrices	$\boldsymbol{\sigma}_1=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$ $\boldsymbol{\sigma}_2=\begin{pmatrix}0 & -\mathbf{i}\\ \mathbf{i} & 0\end{pmatrix}$ $\boldsymbol{\sigma}_3=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$ $\mathbf{1}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$	(2.94)	$\boldsymbol{\sigma}_i$ $\mathbf{1}$ $\mathbf{i}$	Pauli spin matrices 2×2 unit matrix $\mathbf{i}^2=-1$
Anticommutation	$\boldsymbol{\sigma}_i\boldsymbol{\sigma}_j+\boldsymbol{\sigma}_j\boldsymbol{\sigma}_i=2\delta_{ij}\mathbf{1}$	(2.95)	δ_{ij}	Kronecker delta
Cyclic permutation	$\boldsymbol{\sigma}_i\boldsymbol{\sigma}_j=\mathbf{i}\boldsymbol{\sigma}_k$ $(\boldsymbol{\sigma}_i)^2=\mathbf{1}$	(2.96) (2.97)		

Rotation matrices[a]

Rotation about x_1	$\mathbf{R}_1(\theta)=\begin{pmatrix}1 & 0 & 0\\ 0 & \cos\theta & \sin\theta\\ 0 & -\sin\theta & \cos\theta\end{pmatrix}$	(2.98)	$\mathbf{R}_i(\theta)$ θ	matrix for rotation about the ith axis rotation angle
Rotation about x_2	$\mathbf{R}_2(\theta)=\begin{pmatrix}\cos\theta & 0 & -\sin\theta\\ 0 & 1 & 0\\ \sin\theta & 0 & \cos\theta\end{pmatrix}$	(2.99)		
Rotation about x_3	$\mathbf{R}_3(\theta)=\begin{pmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{pmatrix}$	(2.100)	α β γ	rotation about x_3 rotation about x_2' rotation about x_3''
Euler angles			$\mathbf{R}$	rotation matrix

$$\mathbf{R}(\alpha,\beta,\gamma)=\begin{pmatrix}\cos\gamma\cos\beta\cos\alpha-\sin\gamma\sin\alpha & \cos\gamma\cos\beta\sin\alpha+\sin\gamma\cos\alpha & -\cos\gamma\sin\beta\\ -\sin\gamma\cos\beta\cos\alpha-\cos\gamma\sin\alpha & -\sin\gamma\cos\beta\sin\alpha+\cos\gamma\cos\alpha & \sin\gamma\sin\beta\\ \sin\beta\cos\alpha & \sin\beta\sin\alpha & \cos\beta\end{pmatrix} \quad (2.101)$$

[a]Angles are in the right-handed sense for rotation of axes, or the left-handed sense for rotation of vectors. i.e., a vector $\boldsymbol{v}$ is given a right-handed rotation of θ about the x_3-axis using $\mathbf{R}_3(-\theta)\boldsymbol{v}\mapsto\boldsymbol{v}'$. Conventionally, $x_1\equiv x$, $x_2\equiv y$, and $x_3\equiv z$.